2.2 SIMULATION OF INVENTORY SYSTEMS

An important class of simulation problems involves inventory systems. A simple inventory system is shown in Figure 2.11. This inventory system has a periodic review of length N, at which time the inventory level is checked. An order is made to bring the inventory up to the level M. At the end of the first review period, an order quantity, Q_1, is placed. In this inventory system, the lead time (i.e., the length of time between the placement and receipt of an order) is zero. Demands are not usually known with certainty, so the order quantities are probabilistic. Demand is shown as being uniform over the time period in Figure 2.11. In actuality, demands are not usually uniform and do fluctuate over time. One possibility is that demands all occur at the beginning of the cycle. Another is that the lead time is random of some positive length.

Notice that, in the second cycle, the amount in inventory drops below zero, indicating a shortage. In Figure 2.11, these units are backordered; when the order arrives, the demand for the backordered items is satisfied first. To avoid shortages, a buffer, or safety, stock would need to be carried.

Carrying stock in inventory has an associated cost attributed to the interest paid on the funds borrowed to buy the items (this also could be considered as the loss from not having the funds available for other investment purposes). Other costs can be placed in the carrying or holding cost column: renting of storage space, hiring of guards, and so on. An alternative to carrying high inventory is to make more frequent reviews and, consequently, more frequent purchases or replenishments. This has an associated cost: the ordering cost. Also, there is a cost in being short. Customers could get angry, with a subsequent loss of good will. Larger inventories decrease the

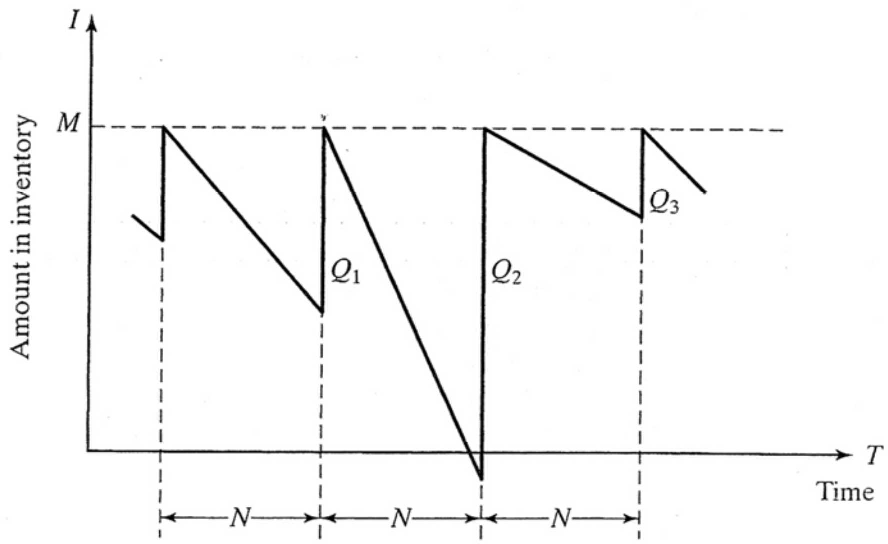


Figure 2.11 Probabilistic order-level inventory system.

Table 2.15 Distribution of Newspapers Demanded Per Day

	Demand Probability Distribution		
Demand	*Good*	*Fair*	*Poor*
40	0.03	0.10	0.44
50	0.05	0.18	0.22
60	0.15	0.40	0.16
70	0.20	0.20	0.12
80	0.35	0.08	0.06
90	0.15	0.04	0.00
100	0.07	0.00	0.00

possibilities of shortages. These costs must be traded off in order to minimize the total cost of an inventory system.

The total cost (or total profit) of an inventory system is the measure of performance. This can be affected by the policy alternatives. For example, in Figure 2.11, the decision maker can control the maximum inventory level, M, and the length of the cycle, N. What effect does changing N have on the various costs?

In an (M, N) inventory system, the events that may occur are the demand for items in the inventory, the review of the inventory position, and the receipt of an order at the end of each review period. When the lead time is zero, as in Figure 2.11, the last two events occur simultaneously.

In the following example for deciding how many newspapers to buy, only a single time period of specified length is relevant, and only a single procurement is made. Inventory remaining at the end of the single time period is sold for scrap or discarded. A wide variety of real-world problems are of this form, including the stocking of spare parts, perishable items, style goods, and special seasonal items [Hadley and Whitin, 1963].

Example 2.3: The News Dealer's Problem

A classical inventory problem concerns the purchase and sale of newspapers. The newsstand buys the papers for 33 cents each and sells them for 50 cents each. Newspapers not sold at the end of the day are sold as scrap for 5 cents each. Newspapers can be purchased in bundles of 10. Thus, the newsstand can buy 50, 60, and so on. There are three types of newsdays: "good"; "fair"; and "poor"; they have the probabilities 0.35, 0.45, and 0.20, respectively. The distribution of newspapers demanded on each of these days is given in Table 2.15. The problem is to compute the optimal number of papers the newsstand should purchase. This will be accomplished by simulating demands for 20 days and recording profits from sales each day.

The profits are given by the following relationship:

$$\text{Profit} = \begin{pmatrix}\text{revenue}\\ \text{from sales}\end{pmatrix} - \begin{pmatrix}\text{cost of}\\ \text{newspapers}\end{pmatrix} - \begin{pmatrix}\text{lost profit from}\\ \text{excess demand}\end{pmatrix} + \begin{pmatrix}\text{salvage from sale}\\ \text{of scrap papers}\end{pmatrix}$$

From the problem statement, the revenue from sales is 50 cents for each paper sold. The cost of newspapers is 33 cents for each paper purchased. The lost profit from excess demand is 17 cents for each paper demanded that could not be provided. Such a shortage cost is somewhat

Table 2.16 Random Digit Assignment for Type of Newsday

Type of Newsday	*Probability*	*Cumulative Probability*	*Random Digit Assignment*
Good	0.35	0.35	01–35
Fair	0.45	0.80	36–80
Poor	0.20	1.00	81–00

Table 2.17 Random Digit Assignments for Newspapers Demanded

	Cumulative Distribution			*Random Digit Assignment*		
Demand	*Good*	*Fair*	*Poor*	*Good*	*Fair*	*Poor*
40	0.03	0.10	0.44	01–03	01–10	01–44
50	0.08	0.28	0.66	04–08	11–28	45–66
60	0.23	0.68	0.82	09–23	29–68	67–82
70	0.43	0.88	0.94	24–43	69–88	83–94
80	0.78	0.96	1.00	44–78	89–96	95–00
90	0.93	1.00	1.00	79–93	97–00	
100	1.00	1.00	1.00	94–00		

controversial, but makes the problem much more interesting. The salvage value of scrap papers is 5 cents each.

Tables 2.16 and 2.17 provide the random digit assignments for the types of newsdays and the demands for those newsdays. To solve this problem by simulation requires setting a policy of buying a certain number of papers each day, then simulating the demands for papers over the 20-day time period to determine the total profit. The policy (number of newspapers purchased) is changed to other values and the simulation repeated until the best value is found.

The simulation table for the decision to purchase 70 newspapers is shown in Table 2.18.

On day 1, the demand is for 80 newspapers, but only 70 newspapers are available. The revenue from the sale of 70 newspapers is \$35.00. The lost profit for the excess demand of 10 newspapers is \$1.70. The profit for the first day is computed as follows:

$$\text{Profit} = \$35.00 - \$23.10 - \$1.70 + 0 = \$10.20$$

On the fourth day, the demand is less than the supply. The revenue from sales of 50 newspapers is \$25.00. Twenty newspapers are sold for scrap at \$0.05 each yielding \$1.00. The daily profit is determined as follows:

$$\text{Profit} = \$25.00 - \$23.10 - 0 + \$1.00 = \$2.90$$

The profit for the 20-day period is the sum of the daily profits, \$131.00. It can also be computed from the totals for the 20 days of the simulation as follows:

$$\text{Total profit} = \$600.00 - \$462.00 - \$17.00 + \$10.00 = \$131.00$$

Table 2.18 Simulation Table for Purchase of 70 Newspapers

Day	*Random Digits for Type of Newsday*	*Type of Newsday*	*Random Digits for Demand*	*Demand*	*Revenue from Sales*	*Lost Profit from Excess Demand*	*Salvage from Sale of Scrap*	*Daily Profit*
1	58	Fair	93	80	$35.00	$1.70	–	$10.20
2	17	Good	63	80	35.00	1.70	–	10.20
3	21	Good	31	70	35.00	–	–	11.90
4	45	Fair	19	50	25.00	–	1.00	2.90
5	43	Fair	91	80	35.00	1.70	–	10.20
6	36	Fair	75	70	35.00	–	–	11.90
7	27	Good	84	90	35.00	3.40	–	8.50
8	73	Fair	37	60	30.00	–	0.50	7.40
9	86	Poor	23	40	20.00	–	1.50	- 1.60
10	19	Good	02	40	20.00	–	1.50	-1.60
11	93	Poor	53	50	25.00	–	1.00	2.90
12	45	Fair	96	80	35.00	1.70	–	10.20
13	47	Fair	33	60	30.00	–	0.50	7.40
14	30	Good	86	90	35.00	3.40	–	8.50
15	12	Good	16	60	30.00	–	0.50	7.40
16	41	Fair	07	40	20.00	–	1.50	- 1.60
17	65	Fair	64	60	30.00	–	0.50	7.40
18	57	Fair	94	80	35.00	1.70	–	10.20
19	18	Good	55	80	35.00	1.70	–	10.20
20	98	Poor	13	40	20.00	–	1.50	- 1.60
					$600.00	$17.00	$10.00	$131.00

where the cost of newspapers for 20 days is $(20 \times \$0.33 \times 70) = \$462.00$. In general, because the results of one day are independent of previous days, inventory problems of this type are easier than queueing problems when solved in a spreadsheet such as is shown in www.bcnn.net and discussed shortly.

Figure 2.12 shows the result of 400 trials, each of twenty days, with a policy of purchasing 70 newspapers per day. For these trials, the average total (20-day) profit was $137.61. The minimum 20-day profit was $64.70 and the maximum was $186.10. Figure 2.12 shows that only 45 of the 400 trials resulted in a total 20-day profit of more than $160.

The manual solution shown in Table 2.18 had a profit of $131.00. This one 20-day is not far from the average over the 400 trials, $137.61; but the result for one 20-day simulation could have been the minimum value or the maximum value. Such an occurrence demonstrates the usefulness of conducting many trials.

On the One Trial sheet, look at the Daily Profit that results when clicking the button 'Generate New Trial'. The results vary quite a bit both in the histogram called 'Frequency of Daily Profit' (showing what happened on each of the 20 days) and in the total profits for those 20 days. The histograms are almost like snowflakes, in that no two are alike! The first two histograms generated are shown in Figure 2.13.

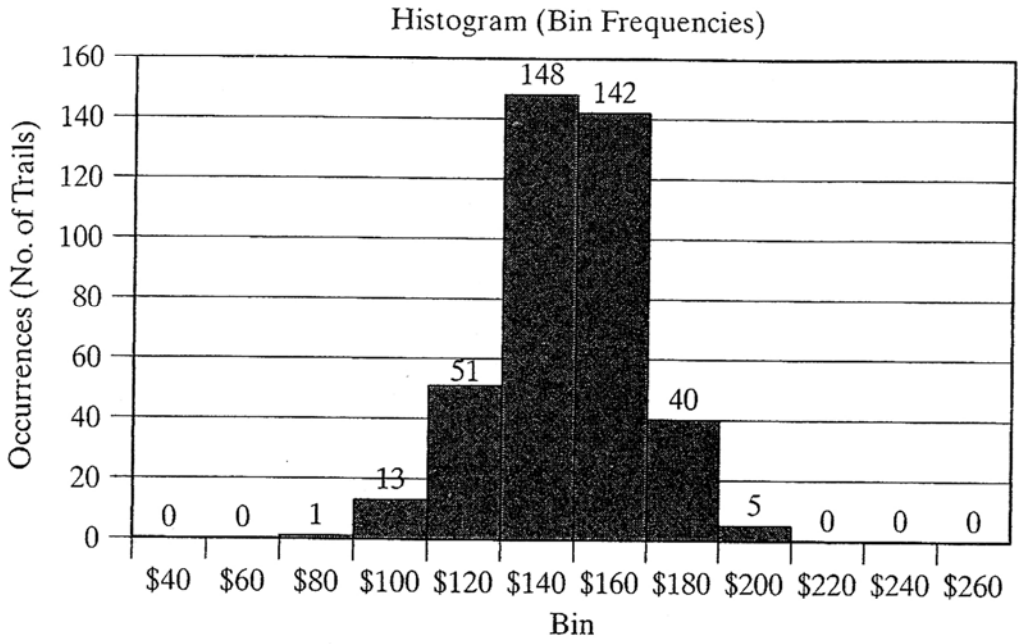
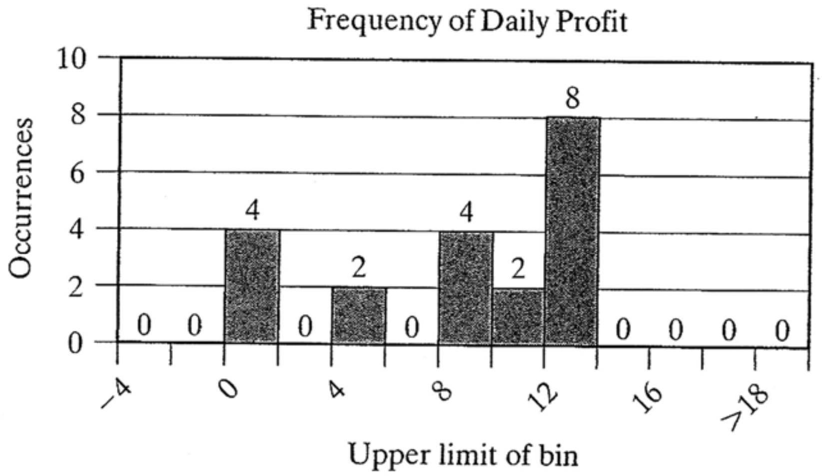


Figure 2.12 Frequency of total (20-day) profits with purchasing of 70 newspapers per day.



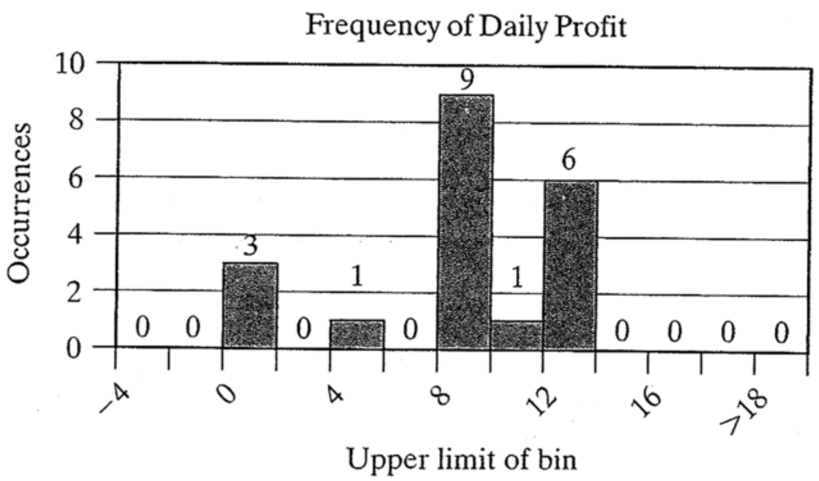


Figure 2.13 First two histograms of daily profit.